**Suslov GDMPK**

**Q1D DMPK fails because**

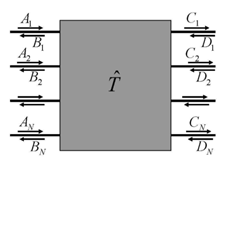
Mentions that the DMPK contains one parameter – the system size in terms of localization length L/ξ, which is true since I believe the one parameter is s/(N+1) = L/ℓ(N+1) = L/ξ. It is equivalent to the super-symmetric sigma model (perhaps check out reference), but allows working with probability distributions (why doesn’t σ model?). Also mentions that Q1D fails even in strong localization since its smallest localization length is order Nℓ (it’s the smallest which determines conductance). But it should be (as in 3D too) of order just ℓ (like it is in 1D). Note that our GDMPK would necessarily reduce to the DMPK version in the limit of 1D (one channel).

**But a 3D DMPK should be possible because**

Makes assertion that we can describe the Anderson transition with weak disorder as long as we’re working near the band edge (aren’t we always?). Elaborates a little…

**General setup for formulation of 3D DMPK**

Moving to describing the scattering matrix…He seems to want to consider the conductor as being connected to N 1-D leads, each of



which carries a propagating electron wave Aneik\_n\*z + Bne-ik\_n\*z on one side, and Cneik\_n\*z + Dne-ik\_n\*z on the other. Is this a serious restriction? Would seem that channels cannot mix (physically)?

***Interpretation of the 3 matrices***

He comments that the canonical representation of the transfer matrix can be interpreted as (going from right to left) (1) mixing channels w/o reflection (υ) (2) reflecting/transmitting along channels (λ) (3) mixing channels again, without reflection (u):

(This above is ‘me’). This is like how Mello interpreted these quantities too. Note that TRS is what requires (my) u2 = u1\* and υ2 = υ1\*.

***Proposed ΔM Statistical Model***

Anyway….the total matrix ΔM he proposes be modelled as:



where ε are small matrices satisfying <εk> = 0, and <εkεkʹ> = αΔL,



and wi are matrices close to 1 (how close? O(δL)? not unitary?). How does this compare to standard model?



where Δ’s are O(√δL).

***Convolution to get incremental changes in eigenvalues, δλ***

Let’s form the product:



Further, we could write:



Now consider the eigenvalues of T. Then he says that the w’s have a negligible effect on the υ’s. So we may say:



Then we examine T12T12†: it’s eigenvalues are λ. So,



And so eigenvalues will be those of the matrix below:



So the perturbation is:



***Generating the equation his way***

Well, he starts with:



What I would do from here, is:



and so arrive at, in a mean field sense,



But he does something different. He says:



Then he inverts the δ function, probably using reversion of series:



And then we’d have:



and then,



And so finally get something like,



And finally, he says we find:



where,



with following definition of matrices:



Let’s work out F and G in more detail,

**Simplifications**

In the meantime, let’s proceed with his simplifications … I will add in his term in red/blue.



and for TRS



So there are two troublesome υ terms, which need to be diagonal to get rid of those √’s.

**Diagonal Approximation**

Basically he says to assume that all the υ-combinations are diagonal. And to assume |υ1mn|2 = |υ2mn|2. These assumptions are consistent with our approximations, but we’d go further and presume that any non-conjugate pair things will also go to zero. Why should the terms be diagonal? Consider the first, and put it in terms of phases.



Seems to me that phases could have non-uniform distribution, in which case there is no reason to set m = n. Or they could have uniform distribution in which case we need φ1 = φ2, really. And in TRS case, we’d have:



Again, I don’t see reason for diagonal rule, m = n. And then the other term is:



I can see that if m = n, then we’ll get sin2(φ1 – φ2), which should be a positive contribution, and this contribution wouldn’t be picked up if we had just matched modulus of matched modulus of υ1’s and υ2’s separately. But then in TRS case,



Can see why we’d want m = n again. There are other arguments for the diagonal approximation….so he pivots to the Landauer formula. The formula for a sample’s conductance depends on how one connects the leads. He distinguishes the formulas g = T and g = T/(1-T) with respect to how the leads/contact resistance, etc., is applied/considered. The first evidentally takes into account the leads’ contact resistance, whereas the second does not [which he seems to prefer because it is measuring an intrinsic property of the conductor, and, does give the proper ∞ value of conductance in the perfect transmission limit].

So while latter definition is better, it is the former which has been used to succesfully explain UCF, and other experimentally observed things. So what physical scenario technically starts with the g = T/(1-T) formula, in principal, but effectively ends up with g = T formula? Well these two formulas match up in the limit of small T. So then question is, how does a metal, for instance have a small T. And answer is perhaps we need to include the ‘skin resistance’ of the metal? This would mean that a true model of a sample must account for the skin resistance. And so then the isotropic model stuff that DMPK used prior would’ve had to have implicitly included this skin resistance term. Perhaps it is this skin resistance that initially scatters the current into an isotropic distribution. Note how in our dK/dz model, it seems that only if you start at γ = 1, can you continue to get it. Otherwise, you’d flow to γ = 0. Perhaps it is the skin resistance which does this? Well, he proposes to simply add two transfer matrices to either side.



[so we’re modeling



]

comparing to the polar form,



we find the following identifications:



In the large λ limit, we can replace 1+λ with λ, and then these equations imply,



In the TRS case, this would mean the matrices are symmetric and anti-symmetric respectively. If we plug this into Aij we get 0, so we’ve taken the limit too far. Rather, let’s write:



where h is Hermitian, and small in some sense. Now if we plug this into the red term, we’d get:



And then,



and then,



So then it would seem natural that, if h statistics are independent of υ, to this order, that i must equal j. And then let’s consider D.



If we were to assume that h is on average zero, then we’d have:



and then we’d have to assume the υik is only correlated with υik, or υik\* as well. In this case we’d have D is diagonal. That’s reaching a bit. But wait, didn’t i do that expansion wrong. Didn’t it assume h was diagonal? And should we have rather that:



And then had:



which would then be:



The zeroth order term goes away,



Let’s say <h> = 0 and <habhaʹbʹ> is diagonal. Then,



and,



and then,



Nothing seems to make i = j.

**Continuing with Diagonal Approximation I**



or for TRS,



Then we make some definitions:



where i, j could be anything. This will reduce to our same Kmn in the TRS case. And then use the identity:



we’d get,



So,



or for TRS



Now define for general, or TRS cases



and factor it out,



Defining:



we can write:



But instead of 3/2 in γ, he has ½. So I’m not agreeing here either. Now defining, and observing,



we can say:



and so we have his result, except for the fact that γn should have a value of 1/2, instead of 3/2, in it. This term matters a little bit, since without it, the RPA doesn’t reduce to Muttalib version, but would reduce to γ = ½.

**Continuing with Diagonal Approximation II**

So if we proceed with the approximation he made with the two matrices on either end, then we’d have, according to him, well, let me define his Jacobian, and derivatives w/r to it.



So then we may write:



where,



Can verify that K, as defined, will make γ → 0 under a random phase approximation.

**Further Approximations**

But, at least for the white-noise model, γ is zero. He also says that γ = 0 corresponds to a random-phase approximation. Perhaps the white noise model makes this approximation. So have to investigate this further. So he seems to say that we have 3 kinds of averaging:

(1) assume neither moduli |υ|2 nor phases φ, are uniformly distributed. Then we get γ term.

(2) assume moduli |υ|2 are not uniform, but phases are. Then Ann → 2Knn and we get γ = 0 (according to him, but γ = ½ according to me)

(3) assume both moduli |υ|2 and phases are uniformly random. Then we get DMPK (or not, according to me – have to recheck calculation)

Think I’ll have to take his equation on faith. His simplified result could still follow if he quoted A, B, C, D, E, F, G, etc. terms wrong in paper by mistake. Or maybe somehow I’m making a mistake.

**Lyapunov exponents equation**

Let’s consider his evolution equation for xn = ln(λ). We’d have:



Average would be:



and then making the assumption that |υ1|2 = |υ2|2, and that the [υ’s] is diagonal, and the definition of A,



Now do the usual identity thing,



Note:



For large λn, we have:



Not sure how he’s getting his result.

**Relation to NLσM**

Then he says that the NLσM makes the same assumptions that the DMPK does. Really? Or at least about the phase? (Is there reference deriving DMPK from NLσM – I think there is). Check it out. And so he says that incorporating non-uniform phases goes beyond the NLσM. This makes sense perhaps, because NLσM does use a white noise model.

**Doing equation my way**

Now let’s work out Wmn,



and for TRS:



And now some expectations:



Let’s specialize to TRS:



and,



and so we have:



and for TRS:



Now we’d perturbatively determine the eigenvalues, and their correlations:



So,



Note the sum over the υ’s gives us 1. And so,



and w/r to TRS,



and if we were to assume a RPA, we’d get:



Note this does *not* reduce to my result. And then,



and so we have:



and then for TRS,



If we were to assume RPA, then we’d have:



and this is my result. This ought to produce the following FP equation:



which is:



and for TRS, we have:



Now let’s pass the second derivative onto the P.



and for TRS we’d have:



Does this match his result? No, he has an additional term, Aii/2, multiplying (1+2λi). But in terms of his matrix definitions, my result is:



or in other words,



where,



and,



Note that in 1D, this reduces to:



And this does not match the 1D result either. Hmmm.

**Diagonal Approximation**

Now if do diagonal approximation on A and D, we get:



and then if we also assume B = C = K, then:



Then we’d use identity,



to get:



Filling these into the equation we have:



which we can write as:



**Random Phase Approximation**

This would simply make Aii = 2Kii. This would restore everything to what we’d expect, except for that extra (1+2λi) term.